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Sliding Mode Observer for The Synchronous Machine with Permanent Magnets

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Abstract — The sensors can be very expensive and their integration very complex in certain industrial processes. The greatnesses not measured estimated by means of observers are going to allow us to reduce the production cost by avoiding us placing sensors. In the linear case the observability is classically determined by a condition of rank and the observers for such systems are generally of type Luenberger, on the other hand in the not linear case, the observability is determined of multiple manners but the classic thought drives to a condition of rank with small arrangements. In the nonlinear case the used observers can be nonlinear sliding mode observers used in our application to estimate the speed and the position of the synchronous machine with permanent magnets.

Keywords — Nonlinear systems, observer, sliding mode, the synchronous machine with permanent magnets.

I. Introduction

The purpose implementation of the laws of command based on the nonlinear model of the system requires the knowledge of the complete state vector of the system at every instant. But, in most of the cases, the only accessible greatnesses of the system are the input and output variables, it is necessary that from this informations to reconstruct the state of the model chosen to elaborate the command. Therefore, the idea bases on the use of an observer [1].

An observer is a dynamic system which we can call it a computer sensor, because it is often implanted on computer to reconstitute or estimate in real time the current state of a system, from the measures and inputs available of the system and knowledge in priori of the model. He allows us then to follow the evolution of the state as information about the system [2].

The possibility of reconstituting internal information on the system by means of the available external greatnesses can be useful for several levels [3]:

- The command of the process, which requires very often the knowledge of his internal state.
- The surveillance of the process, through the gaps between the behavior of the observer and that of the process.

For the nonlinear systems, observer synthesis is still an opened problem. One of the classes the most known for the robust observers is the sliding mode observers.

II. Observability and Observer

The Observability of a process is a very important concept in automatic. Indeed, to reconstruct the state and the output of a system, it is necessary to know, in priori, if the variables of state are observable or not.

Generally, for reasons of technical realization, cost, etc., the dimension of the vector of the output is lower than that of the state. For this reason, at the given moment t, the state *x(t)* cannot be algebraically deducted from the output *y(t)* at this moment [1]. On the other hand, under conditions of observability this state can be deducted from the knowledge of inputs and outputs on a past interval of the time.

The purpose of an observer is to supply with a guaranteed precision the current valuation of the state according to inputs and past outputs.

The principal plan of an observer [3] is given in the scheme mentioned in figure 1:

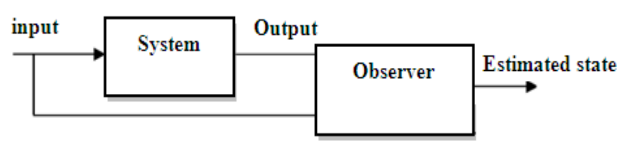

Figure 1. Bloc scheme of an observer

III. Sliding Mode Observer

In most part of the problems of command, the complete state is used in the law of command.

However in the majority of the cases the state is not completely measurable.

To resolve this problem we use an observer to estimate the complete state of the system.

The synthesis of a sliding mode observer consists to force, by means of discontinuous functions, the dynamics of the errors of estimation of a nonlinear system of order n having p outputs to converge on a variety of order *(n - p)* said surface of sliding[4].

The attractiveness and the invariance of the surface of sliding are assured by conditions called conditions of sliding [5].

For a nonlinear system, of the shape:

$$\begin{cases} \dot{x} = f(x,u) \\ y = h(x) \end{cases} \quad (1)$$

A structure of observer by sliding mode written [3]:

$$\begin{cases} \dot{\hat{x}} = \hat{f}(\hat{x},u) + \Lambda sign(y - \hat{y}) \\ \hat{y} = \hat{h}(\hat{x}) \end{cases} \quad (2)$$

It is a copy of the model, to which we add a corrective term, which assures the convergence of $\hat{x}$ to x . The surface of sliding in that case is given by:

$$s(x) = y - \hat{y} \quad (3)$$

The used term of correction is proportional to the discontinuous function sign applied to the error of output where $sign(x)$ is defined by:

$$sign(x) = \begin{cases} 1 & si \quad x > 0 \\ 0 & si \quad x = 0 \\ -1 & si \quad x < 0 \end{cases} \quad (4)$$

IV. Modeling of The Synchronous Machine with Permanent Magnets [6]

The modeling of a synchronous machine with permanent magnets is identical to that of a classic synchronous machine except that the excitement in direct current attached to the rotor is replaced by the flow of the magnet. Thus, the model arises from the model of the classic synchronous machine.

The studied machine is constituted of a stator and a rotor which is characterized by its number of pairs of poles p.

The rolling-ups statoriques are connected in star with isolated neutral.

For a reason of simplification of the modeling we consider the following hypotheses:

- The effect of amortization in the rotor is neglected.
- The magnetic circuit of the machine is not saturated.
- The distribution of the magnetomotive forces is sinusoidal.
- The capacities couplings between the rolling-ups are neglected.
- The phenomena of hysteresis and the currents of Foucault are neglected.
- The irregularities of the air-gap due to statoric notches are ignored.

A. Electric Equations

The equations of the statoriques tensions are:

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = R_s \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \varphi_a \\ \varphi_b \\ \varphi_c \end{bmatrix} \quad (5)$$

With:

$[u_a \; u_b \; u_c]^t$ are the statoric tensions.

R_s is the statoric resistance.

$[i_a \; i_b \; i_c]^t$ are the statoric Currents.

$[\varphi_a \; \varphi_b \; \varphi_c]^t$ are the total statoric flows with:

$$\begin{bmatrix} \varphi_a \\ \varphi_b \\ \varphi_c \end{bmatrix} = [L_{ss}] \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} \varphi_{af} \\ \varphi_{bf} \\ \varphi_{cf} \end{bmatrix} \quad (6)$$

And:

$$\begin{bmatrix} \varphi_{af} \\ \varphi_{bf} \\ \varphi_{cf} \end{bmatrix} = \varphi_f \begin{bmatrix} cos(\theta) \\ cos\left(\theta - \frac{2\pi}{3}\right) \\ cos\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \quad (7)$$

φ_f is the flow leads by magnets in the stator and θ is the angular position of the rotor.

After Park transformation applied for the equations (5) and (6) we obtain the following equation:

$$\begin{cases} u_d = R_s i_d + l_d \frac{di_d}{dt} - l_q w i_q \\ u_q = R_s i_q + l_q \frac{di_q}{dt} + w(l_d i_d + \varphi_f) \end{cases} \quad (8)$$

u_d, u_q, i_d and i_q are the direct components and in quadrature of tension and current.

l_d and l_q are the inductances of direct axis and in quadrature.

φ_f is the flow of magnets through the direct equivalent circuit.

w is the electric speed of the rotor.

B. Mechanical Equations

The mechanical equation of the machine spells:

$$j\frac{d\Omega}{dt} = (c_e - c_r - f\Omega) \quad (9)$$

With:

$\Omega = \frac{w}{p}$ is the mechanical speed of rotation of the machine.

c_e is the electromagnetic couple.

c_r is the resisting couple.

j is the moment of inertia.

p is the number of pairs of poles.

w is the electric speed of the rotor.

f is the coefficient of viscous friction.

C. Expression of the electromagnetic couple

The electromechanic couple developed by the synchronous machines can be given by the following relation:

$$c_e = p(\varphi_\alpha i_\beta - \varphi_\beta i_\alpha) = p(\varphi_d i_q - \varphi_q i_d) \tag{10}$$

$$c_e = p((l_d - l_q)i_d + \varphi_f)i_q \tag{11}$$

If the machine has smooth poles $(\boldsymbol{l_d = l_q})$ we obtain:

$$\boldsymbol{c_e = p\varphi_f i_q} \tag{12}$$

By combination of the electric equations and the mechanical equations, the complete model of the synchronous machine with permanent magnets can be obtained.

D. State model in the rotating mark (d - q)

The not linear model of state in the rotating mark d-q is described by the system below:

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{\Omega} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{l_d} i_d + \frac{p l_q}{l_d} i_q \Omega \\ \frac{-R_s}{l_q} i_q - \frac{p l_d}{l_q} i_d \Omega - \frac{p\varphi_f}{l_q}\Omega \\ \frac{p\varphi_f}{j} i_q - \frac{p(l_q - l_d)}{j} i_d i_q - \frac{f}{j}\Omega \\ \Omega \end{bmatrix} + \begin{bmatrix} \frac{1}{l_d} & 0 & 0 \\ 0 & \frac{1}{l_q} & 0 \\ 0 & 0 & \frac{-1}{j} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_d \\ u_q \\ c_r \end{bmatrix} \tag{13}$$

If the machine has smooth poles $(\boldsymbol{i_d = i_q = i_s})$ we obtain

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_q \\ \dot{\Omega} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{l_s} i_d + p i_q \Omega \\ \frac{-R_s}{l_s} i_q - p i_d \Omega - \frac{p\varphi_f}{l_s}\Omega \\ \frac{p\varphi_f}{j} i_q - \frac{f}{j}\Omega \\ \Omega \end{bmatrix} + \begin{bmatrix} \frac{1}{l_s} & 0 & 0 \\ 0 & \frac{1}{l_s} & 0 \\ 0 & 0 & \frac{-1}{j} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_d \\ u_q \\ c_r \end{bmatrix} \tag{14}$$

E. State model in the fixed mark $(\alpha - \beta)$

The not linear model of state in the fixed mark $(\boldsymbol{\alpha - \beta})$ is described by the system below:

$$\begin{bmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \end{bmatrix} = \frac{A_\theta}{D}\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} - \frac{R_s A_\theta}{D}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} - \frac{2 l_1 w B_\theta}{D}\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} - \frac{w\varphi_f(l_0 + l_1)}{D}\begin{bmatrix} -sin\theta_e \\ cos\theta_e \end{bmatrix} \tag{15}$$

With:

$$l_0 = \frac{l_d + l_q}{2} \tag{16}$$

$$l_1 = \frac{l_d - l_q}{2} \tag{17}$$

$$l_{\alpha\beta} = l_1 sin2\theta_e \tag{18}$$

$$l_\alpha = l_0 + l_1 cos2\theta_e \tag{19}$$

$$l_\beta = l_0 - l_1 cos2\theta_e \tag{20}$$

$$A_\theta = \begin{bmatrix} l_\beta & -l_{\alpha\beta} \\ -l_{\alpha\beta} & l_\alpha \end{bmatrix} \tag{21}$$

$$B_\theta = \begin{bmatrix} -l_a & l_b \\ l_b & l_a \end{bmatrix} \tag{22}$$

$$l_a = l_0 sin2\theta_e \tag{23}$$

$$l_b = l_1 + l_0 cos2\theta_e \tag{24}$$

$$D = |A_\theta| = l_\alpha l_\beta - (l_{\alpha\beta})^2 \tag{25}$$

If the machine has smooth poles we obtain:

$$\begin{bmatrix} \dot{i}_\alpha \\ \dot{i}_\beta \\ \dot{\Omega} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{-R_s}{l_s} i_\alpha - \frac{e_\alpha}{l_s} \\ \frac{-R_s}{l_s} i_\beta - \frac{e_\beta}{l_s} \\ \frac{p\varphi_f}{j}(i_\beta cos\theta_e - i_\alpha sin\theta_e) - \frac{f}{j}\Omega \\ \Omega \end{bmatrix} + \begin{bmatrix} \frac{1}{l_s} & 0 & 0 \\ 0 & \frac{1}{l_s} & 0 \\ 0 & 0 & \frac{-1}{j} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_\alpha \\ u_\beta \\ c_r \end{bmatrix} \tag{26}$$

e_α et e_β are the electromotive forces with :

$$\begin{cases} e_\alpha = -\varphi_f w sin\,\theta_e \\ e_\beta = \varphi_f w cos\,\theta_e \end{cases} \tag{27}$$

V. Construction of The Sliding Mode Observer

This observer is based on the model of synchronous machine with permanent magnets and smooth rotor in a fixed mark expressed by the equation (26).

A sliding mode observer can be then designed as:

$$\begin{cases} \dot{\hat{\imath}}_\alpha = \frac{-R_s}{l_s}\hat{\imath}_\alpha + \frac{p\varphi_f \hat{\Omega} \sin\hat{\theta}_e}{l_s} + \frac{1}{l_s}u_\alpha + K_1 sign(\bar{\imath}_\alpha) \\ \dot{\hat{\imath}}_\beta = \frac{-R_s}{l_s}\hat{\imath}_\beta - \frac{p\varphi_f \hat{\Omega} \cos\hat{\theta}_e}{l_s} + \frac{1}{l_s}u_\beta + K_1 sign(\bar{\imath}_\beta) \\ \dot{\hat{\Omega}} = \frac{p\varphi_f}{j}(\hat{\imath}_\beta cos\hat{\theta}_e - \hat{\imath}_\alpha sin\hat{\theta}_e) - \frac{f_v}{j}\hat{\Omega} + K_2 sign(\bar{\imath}_\alpha) + K_2 sign(\bar{\imath}_\beta) \\ \dot{\hat{\theta}} = \hat{\Omega} \end{cases} \quad (28)$$

With:

$$\bar{\imath}_\alpha = i_\alpha - \hat{\imath}_\alpha \quad (29)$$

$$\bar{\imath}_\beta = i_\beta - \hat{\imath}_\beta \quad (30)$$

K_1 and K_2 are the gains of the observer

The dynamics of the errors are expressed by the following equations:

$$\begin{cases} \dot{\bar{\imath}}_\alpha = \frac{-R_s}{l_s}\bar{\imath}_\alpha + \frac{p\varphi_f(\Omega sin\theta_e - \hat{\Omega}\sin\hat{\theta}_e)}{l_s} - K_1 sign(\bar{\imath}_\alpha) \\ \dot{\bar{\imath}}_\beta = \frac{-R_s}{l_s}\bar{\imath}_\beta + \frac{p\varphi_f(-\Omega cos\theta_e + \hat{\Omega}\cos\hat{\theta}_e)}{l_s} - K_1 sign(\bar{\imath}_\beta) \\ \dot{\bar{\Omega}} = \frac{p\varphi_f}{j}\left[(i_\beta cos\theta_e - i_\alpha sin\theta_e) - (\hat{\imath}_\beta cos\hat{\theta}_e - \hat{\imath}_\alpha sin\hat{\theta}_e)\right] \\ -\frac{f_v}{j}\bar{\Omega} - K_2 sign(\bar{\imath}_\alpha) - K_2 sign(\bar{\imath}_\beta) \\ \dot{\bar{\theta}} = \Omega - \hat{\Omega} \end{cases} \quad (31)$$

With:

$$\bar{\Omega} = \Omega - \hat{\Omega} \quad (32)$$

$$\bar{\theta} = \theta - \hat{\theta} \quad (33)$$

To study the convergence at finished time of our observer, we consider the following function of Lyapunov:

$$V = \frac{1}{2}(\bar{\imath}_\alpha{}^2 + \bar{\imath}_\beta{}^2 + \bar{\Omega}^2 + \bar{\theta}^2) \quad (34)$$

Then:

$$\dot{V} = \bar{\imath}_\alpha\dot{\bar{\imath}}_\alpha + \bar{\imath}_\beta\dot{\bar{\imath}}_\beta + \bar{\Omega}\dot{\bar{\Omega}} + \bar{\theta}\dot{\bar{\theta}} \quad (35)$$

$$\dot{V} = -A_1\bar{\imath}_\alpha{}^2 + A_2\bar{\imath}_\alpha(\Omega sin\theta_e - \hat{\Omega} sin\hat{\theta}_e) - K_1|\bar{\imath}_\alpha| - A_1\bar{\imath}_\beta{}^2 + A_2\bar{\imath}_\beta(\Omega cos\theta_e - \hat{\Omega} cos\hat{\theta}_e) - K_1|\bar{\imath}_\beta| - A_4\bar{\Omega}^2 + A_3\bar{\Omega}\left[(i_\beta cos\theta_e - i_\alpha sin\theta_e) - (\hat{\imath}_\beta cos\hat{\theta}_e - \hat{\imath}_\alpha sin\hat{\theta}_e)\right] - \bar{\Omega}K_2 sign(\bar{\imath}_\alpha) - \bar{\Omega}K_2 sign(\bar{\imath}_\beta) + \bar{\theta}\bar{\Omega} \quad (36)$$

With:

$$A_1 = \frac{R_s}{l_s} \quad (37)$$

$$A_2 = \frac{p\varphi_f}{l_s} \quad (38)$$

$$A_3 = \frac{p\varphi_f}{J} \quad (39)$$

$$A_4 = \frac{f_v}{J} \quad (40)$$

The realistic initial conditions for the group observer-motor are:

$$\left|\Omega sin\theta_e - \hat{\Omega} sin\hat{\theta}_e\right| < 2\Omega_{max} \quad (41)$$

$$|\bar{\imath}_\alpha| = |\bar{\imath}_\beta| < 2i_{max} \quad (42)$$

$$\left|\bar{\imath}_\alpha(\Omega sin\theta_e - \hat{\Omega} sin\hat{\theta}_e)\right| < 4i_{max}\Omega_{max} \quad (43)$$

$$\left|\bar{\imath}_\beta(\Omega cos\theta_e - \hat{\Omega} cos\hat{\theta}_e)\right| < 4i_{max}\Omega_{max} \quad (44)$$

$$\left|\bar{\Omega}\left[(i_\beta cos\theta_e - i_\alpha sin\theta_e) - (\hat{\imath}_\beta cos\hat{\theta}_e - \hat{\imath}_\alpha sin\hat{\theta}_e)\right]\right| < 8i_{max}\Omega_{max} \quad (45)$$

$$\left|\bar{\theta}\bar{\Omega}\right| < 4\theta_{max}\Omega_{max} \quad (46)$$

For:

$$-A_1\bar{\imath}_\alpha{}^2 < 0 \quad (47)$$

$$-A_1\bar{\imath}_\beta{}^2 < 0 \quad (48)$$

$$-A_4\bar{\Omega}^2 < 0 \quad (49)$$

The gains of the observer which assure the convergence of the observer for t> 0 are given by:

$$K_1 > |4a_2\Omega_{max}| \quad (50)$$

$$K_2 > \left|2a_3 i_{max} + \frac{\theta_{max}}{2}\right| \quad (51)$$

VI. Simulation Results

For the validation of this observer in simulation, tests are been realized under Matlab / Simulink and for it, we used the parameters nominal following ones [7]:

Nominal speed=1000 tr per min

Nominal flow (φ_f) =0.1564wb

Statoric resistance (R_s) =0.45 Ω.

Statoric inductance (l_s)=6mh

Motor inertia =0.00176 kg.m^2

Viscous friction (f_v)= 0.0003881Nm/s

Number of pairs of poles (p)=3

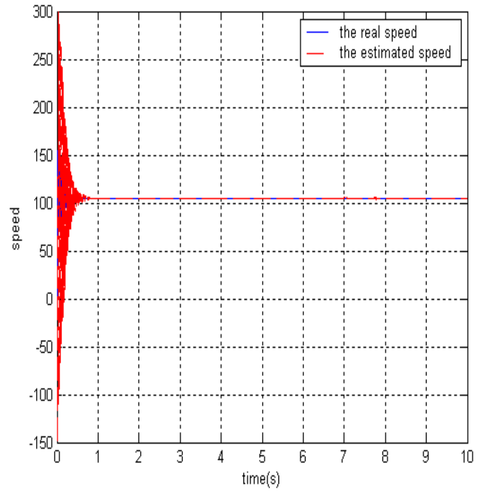

Figure 2. The real and estimated speed

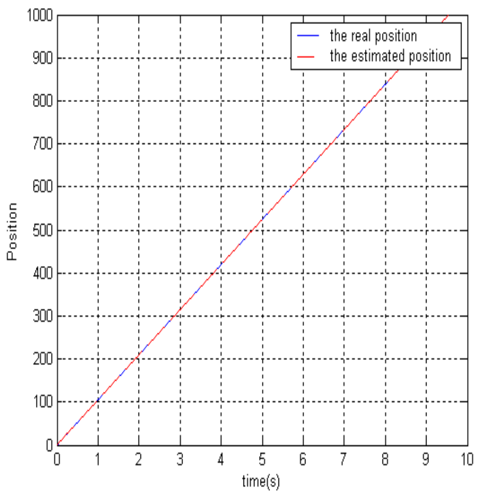

Figure 3. The real and estimated position

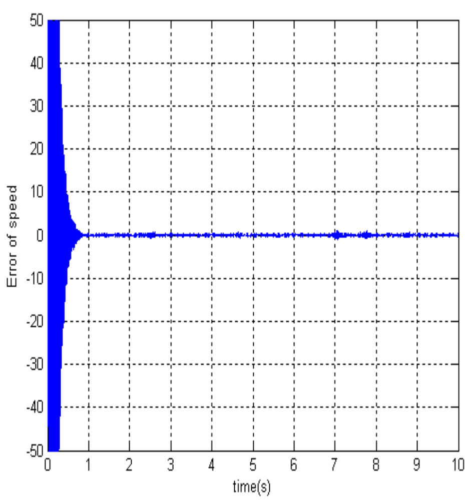

Figure 4. The error of speed

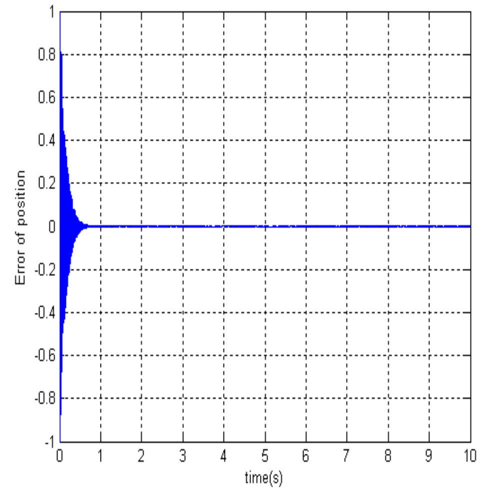

Figure 5. The error of position

The figure 2 shows the measured speed and the speed observed through a sliding mode observer obtained from the not linear model of the synchronous machine with permanent magnets with smooth rotor in the fixed mark *(α-β)*.

The figure 3 shows clearly the error of observation of the speed of the machine, and informs us at the time of convergence of the observer $t<1s$.

The figure 4 shows the measured position and the position observed through the proposed sliding mode observer.

The figure 5 shows clearly the error of observation of the position of the machine with a time of convergence equal to 0.5s.

The results of the simulation obtained show clearly the efficiency and the robustness of the sliding mode observer applied to the synchronous machine with permanent magnets with smooth rotor to estimate its speed and its position.

VII. General Conclusion

In this work, we are interested to the realization of a sliding mode observer to determine the dynamic of the speed and position error of observation for a synchronous machine with permanent magnets.

The results of simulation show the efficiency and the robustness of such observer.

We can add to these advantages the cost that we can win for systems where the sensors are very expensive by report to the integration of a sliding mode observer.

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